

1-8

Perimeter, Circumference, and Area

Common Core State Standards

N-Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas . . .

MP 1, MP 3, MP 4, MP 7

Objectives To find the perimeter or circumference of basic shapes
To find the area of basic shapes

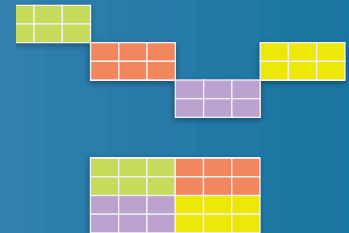


Think about what "wall space" means.



Getting Ready!

You and your friend have two choices for a wall decoration. You say the decoration on the top will use more wall space. Your friend says the two decorations will use the same amount of wall space. Who is correct? Explain.



MATHEMATICAL PRACTICES

In the Solve It, you considered various ideas of what it means to take up space on a flat surface.

Essential Understanding Perimeter and area are two different ways of measuring geometric figures.

The **perimeter** P of a polygon is the sum of the lengths of its sides. The **area** A of a polygon is the number of square units it encloses. For figures such as squares, rectangles, triangles, and circles, you can use formulas for perimeter (or *circumference* C for circles) and area.



Lesson Vocabulary

- perimeter
- area

Take note

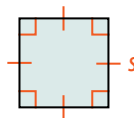
Key Concept Perimeter, Circumference, and Area

Square

side length s

$$P = 4s$$

$$A = s^2$$

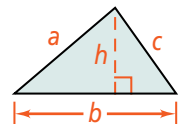


Triangle

side lengths a , b , and c ,
base b , and height h

$$P = a + b + c$$

$$A = \frac{1}{2}bh$$

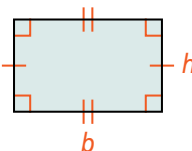


Rectangle

base b and height h

$$P = 2b + 2h, \text{ or } 2(b + h)$$

$$A = bh$$

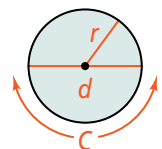


Circle

radius r and diameter d

$$C = \pi d, \text{ or } C = 2\pi r$$

$$A = \pi r^2$$



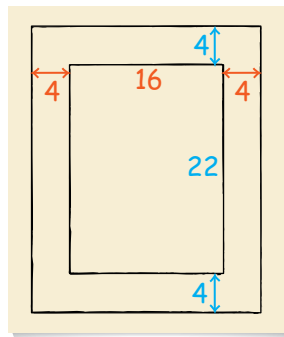
The units of measurement for perimeter and circumference include inches, feet, yards, miles, centimeters, and meters. When measuring area, use square units such as square inches (in.²), square feet (ft²), square yards (yd²), square miles (mi²), square centimeters (cm²), and square meters (m²).



Problem 1 Finding the Perimeter of a Rectangle

Landscaping The botany club members are designing a rectangular garden for the courtyard of your school. They plan to place edging on the outside of the path. How much edging material will they need?

Step 1 Find the dimensions of the garden, including the path.



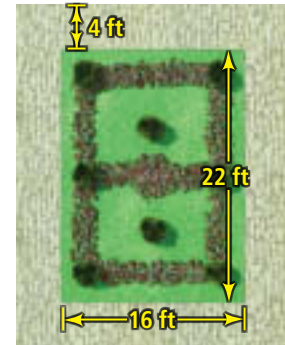
For a rectangle, “length” and “width” are sometimes used in place of “base” and “height.”

Width of the garden and path

$$= 4 + 16 + 4 = 24$$

Length of the garden and path

$$= 4 + 22 + 4 = 30$$



Step 2 Find the perimeter of the garden including the path.

$$\begin{aligned} P &= 2b + 2h && \text{Use the formula for the perimeter of a rectangle.} \\ &= 2(24) + 2(30) && \text{Substitute 24 for } b \text{ and 30 for } h. \\ &= 48 + 60 && \text{Simplify.} \\ &= 108 \end{aligned}$$

You will need 108 ft of edging material.



- Got It?** 1. You want to frame a picture that is 5 in. by 7 in. with a 1-in.-wide frame.
- What is the perimeter of the picture?
 - What is the perimeter of the outside edge of the frame?

You can name a circle with the symbol \odot . For example, the circle with center A is written $\odot A$.

The formulas for a circle involve the special number *pi* (π). Pi is the ratio of any circle's circumference to its diameter. Since π is an irrational number,

$$\pi = 3.1415926 \dots,$$

you cannot write it as a terminating decimal. For an approximate answer, you can use 3.14 or $\frac{22}{7}$ for π . You can also use the π key on your calculator to get a rounded decimal for π . For an exact answer, leave the result in terms of π .

Plan

Why should you draw a diagram?

A diagram can help you see the larger rectangle formed by the garden and the path, and which lengths to add together.

Plan

Which formula should you use?

You want to find circumference. Since you know the diameter in part (A), it would be easier to use the circumference formula that involves diameter.



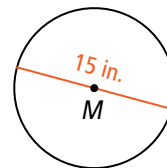
Problem 2 Finding Circumference

What is the circumference of the circle in terms of π ? What is the circumference of the circle to the nearest tenth?

A $\odot M$

$$\begin{aligned} C &= \pi d && \text{Use the formula for circumference of a circle.} \\ &= \pi(15) && \text{This is the exact answer.} \\ &\approx 47.1238898 && \text{Use a calculator.} \end{aligned}$$

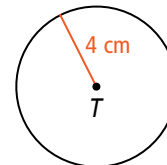
The circumference of $\odot M$ is 15π in., or about 47.1 in.



B $\odot T$

$$\begin{aligned} C &= 2\pi r && \text{Use the formula for circumference of a circle.} \\ &= 2\pi(4) && \text{This is the exact answer.} \\ &= 8\pi && \text{Simplify.} \\ &\approx 25.13274123 && \text{Use a calculator.} \end{aligned}$$

The circumference of $\odot T$ is 8π cm, or about 25.1 cm.



- Got It?** 2. a. What is the circumference of a circle with radius 24 m in terms of π ?
b. What is the circumference of a circle with diameter 24 m to the nearest tenth?

Plan

What do you need?
To find the perimeter of a figure, you need its side lengths. Use what you know about length on a number line and in the coordinate plane.

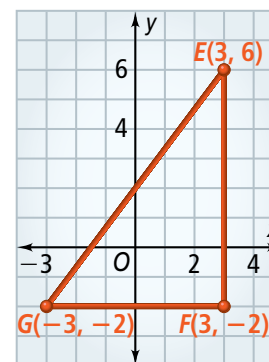


Problem 3 Finding Perimeter in the Coordinate Plane

Coordinate Geometry What is the perimeter of $\triangle EFG$?

Step 1 Find the length of each side.

$$\begin{aligned} EF &= |6 - (-2)| = 8 && \text{Use the Ruler Postulate.} \\ FG &= |3 - (-3)| = 6 \\ EG &= \sqrt{(3 - (-3))^2 + (6 - (-2))^2} && \text{Use the Distance Formula.} \\ &= \sqrt{6^2 + 8^2} && \text{Simplify within the parentheses.} \\ &= \sqrt{36 + 64} && \text{Simplify.} \\ &= \sqrt{100} \\ &= 10 \end{aligned}$$



Step 2 Add the side lengths to find the perimeter.

$$EF + FG + EG = 8 + 6 + 10 = 24$$

The perimeter of $\triangle EFG$ is 24 units.



- Got It?** 3. Graph quadrilateral $JKLM$ with vertices $J(-3, -3)$, $K(1, -3)$, $L(1, 4)$, and $M(-3, 1)$. What is the perimeter of $JKLM$?

To find area, you should use the same unit for both dimensions.



Problem 4 Finding Area of a Rectangle

Banners You want to make a rectangular banner similar to the one at the right. The banner shown is $2\frac{1}{2}$ ft wide and 5 ft high. To the nearest square yard, how much material do you need?

Step 1 Convert the dimensions of the banner to yards. Use the conversion factor $\frac{1 \text{ yd}}{3 \text{ ft}}$.

$$\text{Width: } \frac{5}{2} \text{ ft} \cdot \frac{1 \text{ yd}}{3 \text{ ft}} = \frac{5}{6} \text{ yd} \quad 2\frac{1}{2} \text{ ft} = \frac{5}{2} \text{ ft}$$

$$\text{Height: } 5 \text{ ft} \cdot \frac{1 \text{ yd}}{3 \text{ ft}} = \frac{5}{3} \text{ yd}$$

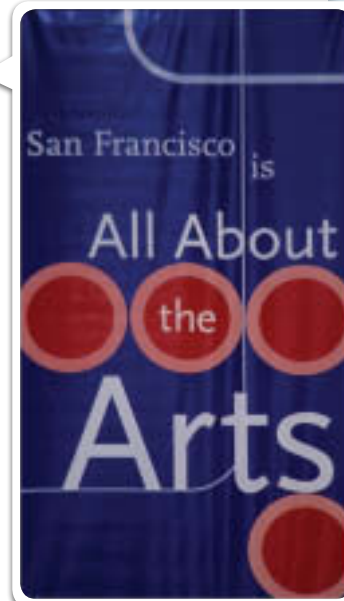
Step 2 Find the area of the banner.

$$A = bh \quad \text{Use the formula for area of a rectangle.}$$

$$= \frac{5}{6} \cdot \frac{5}{3} \quad \text{Substitute } \frac{5}{6} \text{ for } b \text{ and } \frac{5}{3} \text{ for } h.$$

$$= \frac{25}{18}$$

The area of the banner is $\frac{25}{18}$, or $1\frac{7}{18}$ square yards (yd^2). You need 2 yd^2 of material.



Think

How can you check your conversion?

Yards are longer than feet, so the number you get in yards should be less than the given number in feet. Since $\frac{5}{6} < 2\frac{1}{2}$, the conversion checks.



Got It? 4. You are designing a poster that will be 3 yd wide and 8 ft high. How much paper do you need to make the poster? Give your answer in square feet.



Problem 5 Finding Area of a Circle

What is the area of $\odot K$ in terms of π ?

Step 1 Find the radius of $\odot K$.

$$r = \frac{16}{2}, \text{ or } 8 \quad \text{The radius is half the diameter.}$$

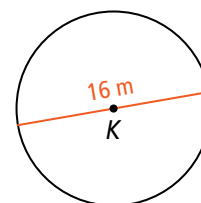
Step 2 Use the radius to find the area.

$$A = \pi r^2 \quad \text{Use the formula for area of a circle.}$$

$$= \pi(8)^2 \quad \text{Substitute 8 for } r.$$

$$= 64\pi \quad \text{Simplify.}$$

The area of $\odot K$ is $64\pi \text{ m}^2$.



Plan

What are you given?

In circle problems, make it a habit to note whether you are given the radius or the diameter. In this case, you are given the diameter.



Got It? 5. The diameter of a circle is 14 ft.

- What is the area of the circle in terms of π ?
- What is the area of the circle using an approximation of π ?
- Reasoning** Which approximation of π did you use in part (b)? Why?

The following postulate is useful in finding areas of figures with irregular shapes.

Take note

Postulate 1-10 Area Addition Postulate

The area of a region is the sum of the areas of its nonoverlapping parts.



Problem 6 Finding Area of an Irregular Shape

Multiple Choice What is the area of the figure at the right?

All angles are right angles.

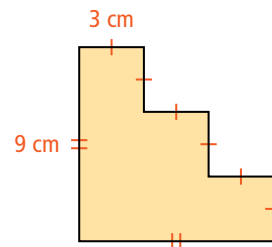
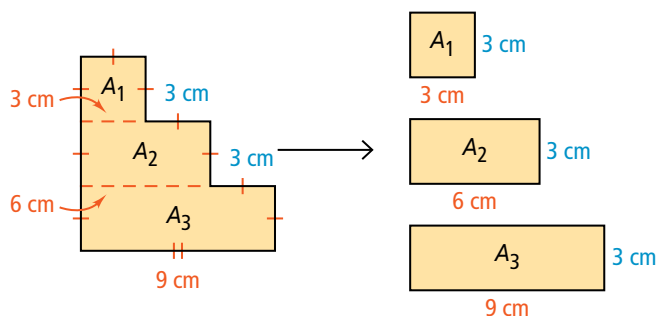
(A) 27 cm^2

(B) 36 cm^2

(C) 45 cm^2

(D) 54 cm^2

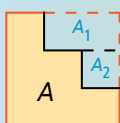
Step 1 Separate the figure into rectangles.



Think

What is another way to find the area?

Extend the figure to form a square. Then subtract the areas of basic shapes from the area of the square.



$$A = A_{\text{square}} - A_1 - A_2$$

Step 2 Find A_1 , A_2 , and A_3 .

$$\text{Area} = bh$$

Use the formula for the area of a rectangle.

$$A_1 = 3 \cdot 3 = 9$$

Substitute for the base and height.

$$A_2 = 6 \cdot 3 = 18$$

$$A_3 = 9 \cdot 3 = 27$$

Step 3 Find the total area of the figure.

$$\text{Total Area} = A_1 + A_2 + A_3$$

Use the Area Addition Postulate.

$$= 9 + 18 + 27$$

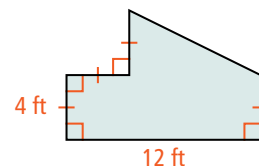
$$= 54$$

The area of the figure is 54 cm^2 . The correct choice is D.



Got It? 6. a. **Reasoning** What is another way to separate the figure in Problem 6?

b. What is the area of the figure at the right?

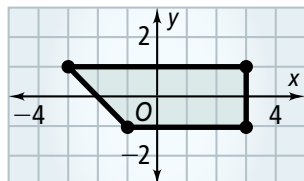




Lesson Check

Do you know HOW?

- What is the perimeter and area of a rectangle with base 3 in. and height 7 in.?
- What is the circumference and area of each circle to the nearest tenth?
 - $r = 9$ in.
 - $d = 7.3$ m
- What is the perimeter and area of the figure at the right?



Do you UNDERSTAND?



- Writing** Describe a real-world situation in which you would need to find a perimeter. Then describe a situation in which you would need to find an area.
- Compare and Contrast** Your friend can't remember whether $2\pi r$ computes the circumference or the area of a circle. How would you help your friend? Explain.
- Error Analysis** A classmate finds the area of a circle with radius 30 in. to be 900 in.^2 . What error did your classmate make?



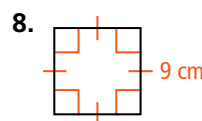
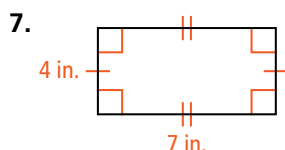
Practice and Problem-Solving Exercises



A Practice

Find the perimeter of each figure.

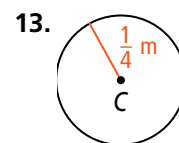
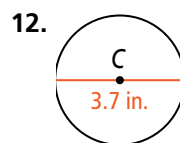
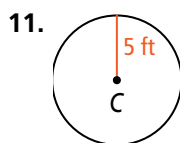
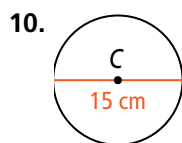
← See Problem 1.



9. **Fencing** A garden that is 5 ft by 6 ft has a walkway 2 ft wide around it. What is the amount of fencing needed to surround the walkway?

Find the circumference of $\odot C$ in terms of π .

← See Problem 2.



Coordinate Geometry Graph each figure in the coordinate plane. Find each perimeter.

← See Problem 3.

14. $X(0, 2)$, $Y(4, -1)$, $Z(-2, -1)$

15. $A(-4, -1)$, $B(4, 5)$, $C(4, -2)$

16. $L(0, 1)$, $M(3, 5)$, $N(5, 5)$, $P(5, 1)$

17. $S(-5, 3)$, $T(7, -2)$, $U(7, -6)$, $V(-5, -6)$

Find the area of each rectangle with the given base and height.

← See Problem 4.

18. 4 ft, 4 in.

19. 30 in., 4 yd

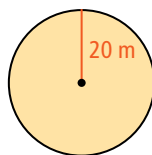
20. 2 ft 3 in., 6 in.

21. 40 cm, 2 m

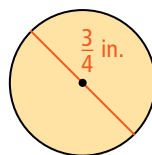
22. **Roads** What is the area of a section of pavement that is 20 ft wide and 100 yd long? Give your answer in square feet.

Find the area of each circle in terms of π .

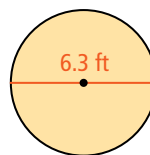
23.



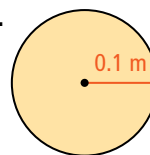
24.



25.



26.



← See Problem 5.

Find the area of each circle using an approximation of π . If necessary, round to the nearest tenth.

27. $r = 7$ ft

28. $d = 8.3$ m

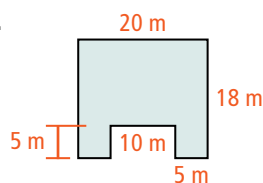
29. $d = 24$ cm

30. $r = 12$ in.

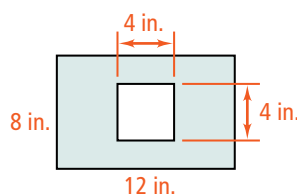
Find the area of the shaded region. All angles are right angles.

← See Problem 6.

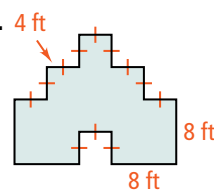
31.



32.



33.



Home Maintenance To determine how much of each item to buy, tell whether you need to know area or perimeter. Explain your choice.

34. wallpaper for a bedroom

35. crown molding for a ceiling

36. fencing for a backyard

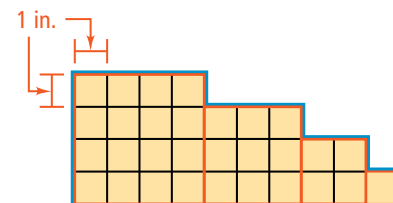
37. paint for a basement floor

38. **Think About a Plan** A light-year unit describes the distance that one photon of light travels in one year. The Milky Way galaxy has a diameter of about 100,000 light-years. The distance to Earth from the center of the Milky Way galaxy is about 30,000 light-years. How many more light-years does a star on the outermost edge of the Milky Way travel in one full revolution around the galaxy compared to Earth?

- What do you know about the shape of each orbital path?
- Are you looking for circumference or area?
- How do you compare the paths using algebraic expressions?

39. a. What is the area of a square with sides 12 in. long? 1 ft long?
b. How many square inches are in a square foot?

40. a. Count squares at the right to find the area of the polygon outlined in blue.
b. Use a formula to find the area of each square outlined in red.
c. **Writing** How does the sum of your results in part (b) compare to your result in part (a)? Which postulate does this support?



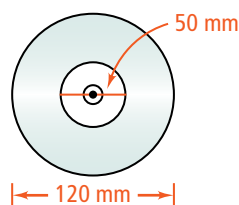
41. The area of an 11-cm-wide rectangle is 176 cm^2 . What is its length?

42. A square and a rectangle have equal areas. The rectangle is 64 cm by 81 cm. What is the perimeter of the square?

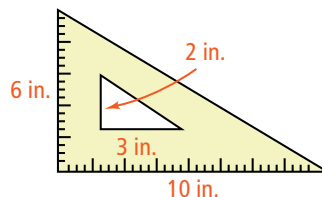
43. A rectangle has perimeter 40 cm and base 12 cm. What is its area?

Find the area of each shaded figure.

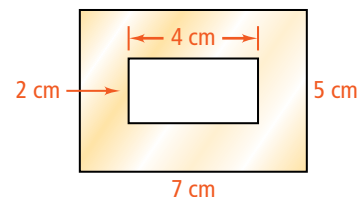
44. compact disc



45. drafting triangle



46. picture frame



47. a. **Reasoning** Can you use the formula for the perimeter of a rectangle to find the perimeter of any square? Explain.
 b. Can you use the formula for the perimeter of a square to find the perimeter of any rectangle? Explain.
 c. Use the formula for the perimeter of a square to write a formula for the area of a square in terms of its perimeter.
48. **Estimation** On an art trip to England, a student sketches the floor plan of the main body of Salisbury Cathedral. The shape of the floor plan is called the building's "footprint." The student estimates the dimensions of the cathedral on her sketch at the right. Use the student's lengths to estimate the area of Salisbury Cathedral's footprint.
49. **Coordinate Geometry** The endpoints of a diameter of a circle are $A(2, 1)$ and $B(5, 5)$. Find the area of the circle in terms of π .
50. **Algebra** A rectangle has a base of x units. The area is $(4x^2 - 2x)$ square units. What is the height of the rectangle in terms of x ?
- (A) $(4 - x)$ units (C) $(4x^3 - 2x^2)$ units
 (B) $(x - 2)$ units (D) $(4x - 2)$ units

Coordinate Geometry Graph each rectangle in the coordinate plane. Find its perimeter and area.

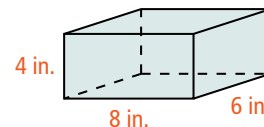
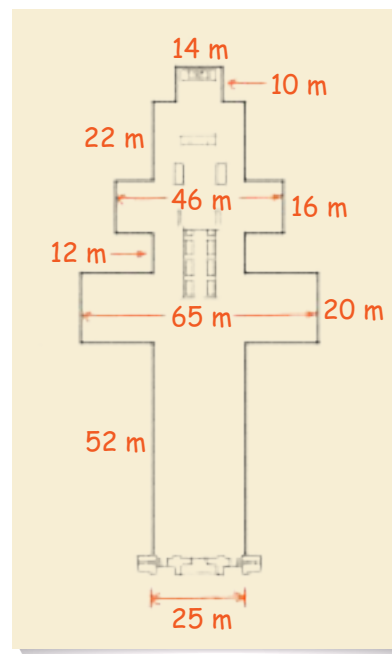
51. $A(-3, 2)$, $B(-2, 2)$, $C(-2, -2)$, $D(-3, -2)$

52. $A(-2, -6)$, $B(-2, -3)$, $C(3, -3)$, $D(3, -6)$

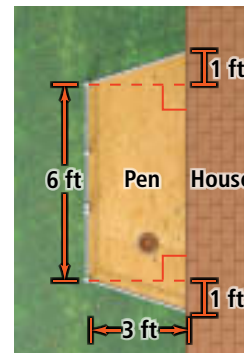
53. The surface area of a three-dimensional figure is the sum of the areas of all of its surfaces. You can find the surface area by finding the area of a net for the figure.

- a. Draw a net for the solid shown. Label the dimensions.
 b. What is the area of the net? What is the surface area of the solid?

54. **Coordinate Geometry** On graph paper, draw polygon $ABCDEFGH$ with vertices $A(1, 1)$, $B(10, 1)$, $C(10, 8)$, $D(7, 5)$, $E(4, 5)$, $F(4, 8)$, and $G(1, 8)$. Find the perimeter and the area of the polygon.



55. **Pet Care** You want to adopt a puppy from your local animal shelter. First, you plan to build an outdoor playpen along the side of your house, as shown on the right. You want to lay down special dog grass for the pen's floor. If dog grass costs \$1.70 per square foot, how much will you spend?
56. A rectangular garden has an 8-ft walkway around it. How many more feet is the outer perimeter of the walkway than the perimeter of the garden?



Challenge **Algebra** Find the area of each figure.

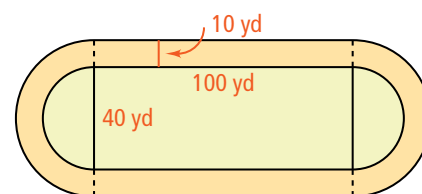
57. a rectangle with side lengths $\frac{2a}{5b}$ units and $\frac{3b}{8}$ units
58. a square with perimeter $10n$ units
59. a triangle with base $(5x - 2y)$ units and height $(4x + 3y)$ units

Standardized Test Prep

GRIDDED RESPONSE

SAT/ACT

60. An athletic field is a 100 yd-by-40 yd rectangle with a semicircle at each of the short sides. A running track 10 yd wide surrounds the field. Find the perimeter of the outside of the running track to the nearest tenth of a yard.
61. A square garden has a 4-ft walkway around it. The garden has a perimeter of 260 ft. What is the area of the walkway in square feet?
62. $A(4, -1)$ and $B(-2, 3)$ are points in a coordinate plane. M is the midpoint of \overline{AB} . What is the length of \overline{MB} to the nearest tenth of a unit?
63. Find CD to the nearest tenth if point C is at $(12, -8)$ and point D is at $(5, 19)$.



Mixed Review

Find (a) AB to the nearest tenth and (b) the midpoint coordinates of \overline{AB} .

← See Lesson 1-7.

64. $A(4, 1), B(7, 9)$

65. $A(0, 3), B(-3, 8)$

66. $A(-1, 1), B(-4, -5)$

\overleftrightarrow{BG} is the perpendicular bisector of \overline{WR} at point K .

← See Lesson 1-6.

67. What is $m\angle BKR$?

68. Name two congruent segments.

Get Ready! To prepare for Lesson 2-1, do Exercise 69.

69. a. Copy and extend this list to show the first 10 perfect squares.

$$1^2 = 1, 2^2 = 4, 3^2 = 9, 4^2 = 16, \dots$$

← See p. 889.

b. Which do you think describes the square of any odd number?

It is odd.

It is even.